3. Homework Assignments

Dynamical Systems II

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http://dynamics.mi.fu-berlin.de/lectures/

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Problem 1: Annalyx plays ideal billiard on a circular table. Which direction should he choose to reach a dense set on the periphery?

Problem 2: Let $0 < \beta < 1$ be irrational and

$$s_n := \text{sign}\,(\sin(n\pi\beta)), \qquad n = 1, 2, 3, ...$$

The sequence

$$w_n := |s_n - s_{n+1}|/2$$

detects the sign changes of the sequence s_n. Prove:

$$\lim_{N\to\infty} \frac{1}{N} \sum_{n=1}^{N} w_n = \beta.$$

Free extra: Is it possible to recover a rational number β from the sequence s_n?

Problem 3: Let $f : S^1 \to S^1$ be a homeomorphism of the circle that reverses orientation, i.e. the induced map $F : \mathbb{R} \to \mathbb{R}$ on the covering space $\mathbb{R}$ satisfies $F(x+2\pi) = F(x) - 2\pi$ for all $x \in \mathbb{R}$.

Prove or disprove: f has a fixed point.
Would it be interesting to introduce a concept of rotation number for orientation reversing homeomorphisms F?

Problem 4: Find an example to show that a homeomorphism f of the circle is not necessarily conjugate to a rigid rotation $g(x) = x + 2\pi\alpha$ with constant α.

(i) by a homeomorphism h,

(ii) by any bijection h of S^1.

(f is conjugate to g by h iff $h \circ f \circ h^{-1} = g$)